LETTERS

Generation of optical 'Schrödinger cats' from photon number states

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Schrödinger's cat[1] is a *Gedankenexperiment* in quantum physics, in which an atomic decay triggers the death of the cat. Because quantum physics allow atoms to remain in superpositions of states, the classical cat would then be simultaneously dead and alive. By analogy, a 'cat' state of freely propagating light can be defined as a quantum superposition of well separated quasi-classical states[2,3]—it is a classical light wave that simultaneously possesses two opposite phases. Such states play an important role in fundamental tests of quantum theory[4–7] and in many quantum information processing tasks, including quantum computation[8], quantum teleportation[9,10] and precision measurements[11]. Recently, optical Schrödinger 'kittens' were prepared[12–14]; however, they are too small for most of the aforementioned applications and increasing their size is experimentally challenging. Here we demonstrate, theoretically and experimentally, a protocol that allows the generation of arbitrarily large squeezed Schrödinger cat states, using homodyne detection and photon number states as resources. We implemented this protocol with light pulses containing two photons, producing a squeezed Schrödinger cat state with a negative Wigner function. This state clearly exhibits several quantum phase-space interference fringes between the 'dead' and 'alive' components, and is large enough to become useful for quantum information processing and experimental tests of quantum theory.

The predictions of quantum physics for microscopic objects cannot be simply generalized to our 'classical' world. In fact, the reason why Schrödinger's cats are so hard to prepare is the same that makes large quantum computers so hard to build: for macroscopic systems, quantum state superpositions rapidly decohere into statistical mixtures because of strong interactions with the environment. To become feasible, Schrödinger's *Gedankenexperiment* should be transposed from a cat to a more convenient physical system, with its own 'classical' or 'quasi-classical' states. In quantum optics, they correspond to coherent states $|\alpha\rangle$, where α is the coherent amplitude[15,16]. Therefore, a quantum superposition $|\psi\rangle = \mathcal{N}(|\alpha\rangle + e^{i\theta}|-\alpha\rangle)$ defines a optical cat state with a 'size' $|\alpha|^2$, where $\mathcal{N} = \left[2\left(1+\cos(\theta)e^{-2|\alpha|^2}\right)\right]^{-1/2}$ is a normalization constant. As the phase origin is arbitrary, we will assume in the following that α is real.

In addition to their numerous applications[4–11], optical cat states have another crucial advantage: quantum optics provide efficient tools to tell the difference between a true quantum superposition and a plain statistical mixture of two coherent states. Quantum states of light, often considered in terms of photons, can also be described as waves, using their amplitudes and phases or, in cartesian coordinates, their quadratures $\hat{x}$ and $\hat{p}$ (ref. 17). A state is then characterized by the quasi-probability distribution of its quadratures $W(x, p)$, called the Wigner function[18]. It can be reconstructed by homodyne tomography from several marginal quadrature distributions $P(\hat{x}_\theta = \hat{x}\cos\theta + \hat{p}\sin\theta)$ measured with a homodyne detection. As $\hat{x}$ and $\hat{p}$ are not simultaneously defined in quantum physics, the Wigner function may become negative for specific quantum states, including optical 'Schrödinger cats'. In this case, the Wigner function clearly reveals the difference between a real quantum superposition and a mere statistical mixture of the two states $|\pm\alpha\rangle$: for a true superposition state, it presents a phase-space interference between the 'dead' ($|-\alpha\rangle$) and 'alive' ($|+\alpha\rangle$) components and takes negative values.

Such superposition states could only be observed in bound systems[19,20] until, very recently, several groups succeeded in preparing free-propagating light beams in small cat states ('Schrödinger kittens')[12–14]. These experiments attracted much attention, as first steps on a new promising way towards quantum communication. But the size of the 'kittens' accessible so far is limited to $|\alpha|^2 \lesssim 1$, and their amplification[21] remains a serious experimental challenge[22]. On the other hand, most quantum information processing applications require larger cats with a smaller overlap between the two coherent states: it should be typically less than 1%, which corresponds to $|\alpha|^2 \gtrsim 2.3$.

In this letter we demonstrate, theoretically and experimentally, a method to produce quantum superpositions of squeezed coherent states with arbitrarily large amplitudes (see Fig. 1). These cat states are squeezed along the x quadrature and stretched along p, which makes them more robust against decoherence[23]. If needed, they can be easily 'un-squeezed', either by injecting them into a degenerate optical parametric amplifier, or by mixing them with squeezed vacuum[24,25]. The required squeezing, around 3 dB, is easily achievable.

The basis of our protocol is to split a photon number state (Fock state) containing exactly n photons on a 50/50 beam splitter (BS), and to measure the momentum quadrature $\hat{p}$ in one mode. The desired state is prepared in the other mode, under the condition that $|p| \le \varepsilon \ll 1$.

An interesting insight into the structure of the prepared state is obtained by looking at its wavefunction ϕ_n, in the limit $\varepsilon \to 0$ (as we show below, a finite ε is a second-order effect that does not perturb our experiments). Omitting the normalization factors, the wave function of a n-photon number state in the momentum quadrature basis is $H_n(p)e^{-p^2/2}$, where H_n is the nth Hermite polynomial. Mixed

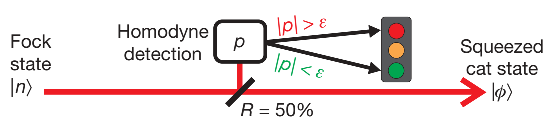

Figure 1 | Preparing squeezed 'Schrödinger cat' states from Fock states using a single homodyne detection. A photon number state containing n photons is divided into two modes on a beam splitter with 50% reflectivity. A homodyne detection measures the momentum quadrature $\hat{p}$ in one mode. If the measurement outcome p is close to 0 within an acceptance width ε ($|p| \le \varepsilon \ll 1$), the other mode is successfully prepared in a 'squeezed cat' state, otherwise it is discarded. See text for details.

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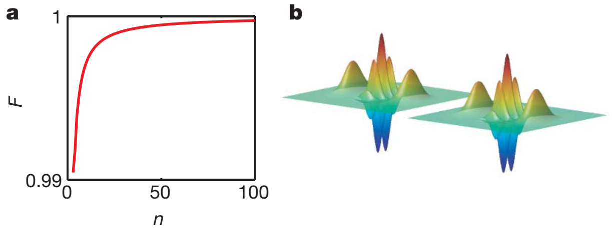

Figure 2 | Theoretical performance. a, Fidelity F between the state produced with n photons and an ideal Schrödinger cat with a 'size' $|\alpha|^2 = n$, squeezed by 3 dB. **b,** Example of ideal state preparation. The Wigner function of the pure state prepared from 10 photons (left) compared with an ideal Schrödinger cat state with $\alpha = \sqrt{10}$ squeezed by 3 dB (right). Their fidelity is $F_{10} \approx 99.7\%$.

with vacuum on a 50/50 BS, the two-mode wavefunction becomes $\tilde{\phi}(p, p_0) = H_n\left[(p - p_0)/\sqrt{2}\right] e^{-(p^2+p_0^2)/2}$. If the measurement outcome is $p_0 = 0$, by taking the Fourier transform we see that the un-normalized wavefunction in the position basis is simply $\phi_n(x) = x^n e^{-x^2/2}$.

For $n \geq 3$, this state has a fidelity $F_n > 99\%$ with a 'Schrödinger cat' state with a size $|\alpha|^2 = n$ and a superposition phase $\theta = n\pi$, which has been squeezed by 3 dB along the x axis. Remarkably enough, the quality of the prepared 'cats' increases with their size, as shown in Fig. 2a. We see numerically that $F_n \approx 1 - 0.03/n$, and we rigorously prove in the Supplementary Information that the fidelity tends to 1 when $n \to \infty$. For small n we observe a slight deviation from this scaling law: when $n = 2$, a cat state with $|\alpha|^2 = 2.6$ squeezed by 3.5 dB is obtained with a 99% fidelity. As an illustration, we present in Fig. 2b the Wigner function of the pure state prepared with 10 photons, compared to an ideal cat state $\mathcal{N}\left(|\sqrt{10}\rangle + |-\sqrt{10}\rangle\right)$ squeezed by 3 dB. In this case the fidelity is $F_{10} \approx 99.7\%$.

We implemented this protocol experimentally using ultrashort light pulses (180 fs) prepared in $n = 2$ number states (see Fig. 3). Their preparation is detailed elsewhere[26]. In brief, two beams containing the same number of photons (two-mode squeezed state) are produced in a spatially non-degenerate optical parametric amplifier (OPA) by down-conversion of frequency-doubled femtosecond laser pulses. One of them is split between two avalanche photodiodes (APDs) after spatial and spectral filtering. A coincidence APD detection heralds the presence of at least two photons, and as the parametric gain is not too large ($g = 1.17$), this projects the other mode in a two-photon number state.

These $n = 2$ states are split on a 50/50 BS. The reflected mode is measured by a time-resolved homodyne detection. We accept the outcome p if $|p| < 0.1$, which leads to a success probability of ~7.5%. This prepares the desired 'squeezed cat' states in the other mode, with a rate of ~7 s^{-1}. To analyse these states, we perform a homodyne tomography with a second detection, measuring six different quadrature distributions with 15,000 data points each. From these distributions, using a maximal-likelihood algorithm, we reconstruct the Wigner function of the prepared state corrected for the losses of the final homodyne detection. We note that the defects of the first detection, involved in the preparation of the state, cannot be compensated.

The reconstructed Wigner function, presented in Fig. 4, is clearly negative. We observe the expected phase-space interference between two coherent states with amplitudes $\alpha = \pm\sqrt{2.6}$ squeezed by 3.5 dB. As shown below, the difference from the ideal 'squeezed cat' is essentially due to technical issues.

The prepared states are very sensitive to experimental imperfections. Dark counts and stray light decrease the probability ξ for an APD detection to correspond to the desired photon number state preparation. Mode distortion in the nonlinear crystals and imperfect laser beams lead to an impure initial two-mode squeezed state. We can consider that the associated excess noise is added by phase-independent parametric amplification with a gain $h = \cosh(\gamma r)^2$ on

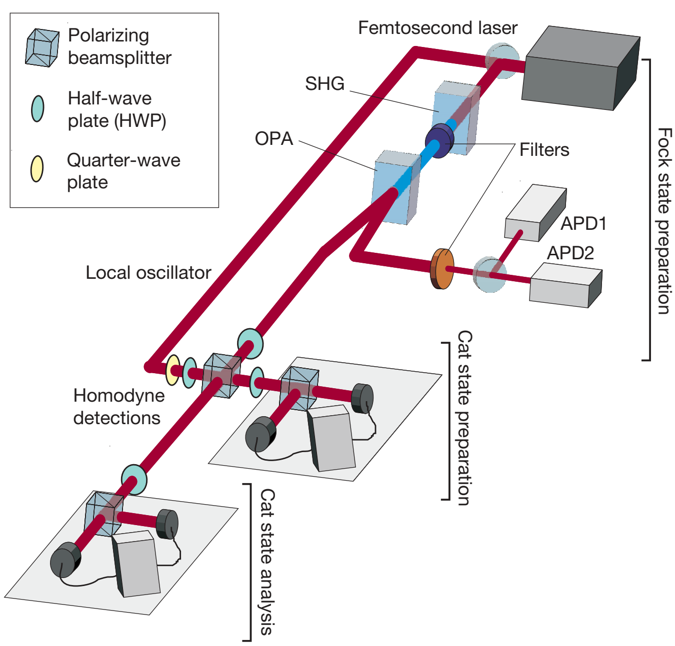

Figure 3 | Experimental set-up. Femtosecond pulses, frequency-doubled by second harmonic generation (SHG), pump a spatially degenerate optical parametric amplifier (OPA). A two-photon state is prepared in one mode by a coincidence detection in the other, using two avalanche photodiodes (APD). It is split between two homodyne detectors: one is used for the preparation of the cat state, the other for the analysis (see text for details).

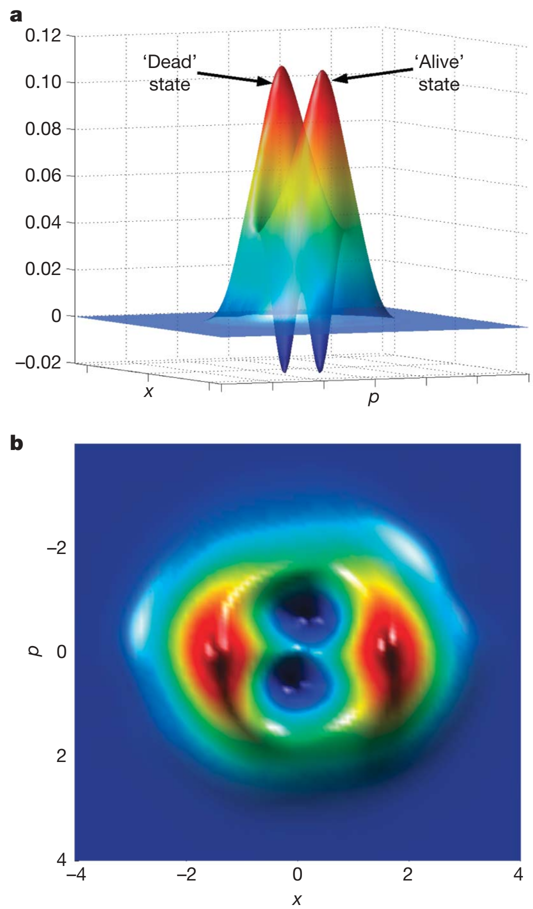

Figure 4 | Experimental results. a, b, Experimental Wigner function $W(x, p)$ produced with $n = 2$ photons, corrected for the losses of the final homodyne detection (**a**, side view; **b**, top view). An interference between the 'dead' and 'alive' states with two negative regions is clearly visible.

a pure two-mode state squeezed by $s=\exp(-2r)$, where γ is the relative efficiency of the amplification process responsible for the excess noise. The limited efficiency η and the noise e of the homodyne detection involved in the state preparation decrease the purity of the final state. The defects of the second detection are not involved in the preparation but only in the analysis of the generated states, and must be corrected for to determine the actual Wigner function.

Taking all these parameters into account, we derived an analytical model for the generated states (see Supplementary Information). Figure 5a presents the Wigner function obtained from our model with the actual values of the experimental parameters, and we see it is extremely similar to the one reconstructed from the data with the maximal-likelihood algorithm. Figure 5b shows the Wigner function we would obtain with pure photon number states and a lossless detection for the same acceptance width $\varepsilon=0.1$, compared to the pure case ($\varepsilon=0$). Their fidelity is 99%, which shows that our experiment is limited by technical issues and not by the finite ε. In a general case, the effect of ε is discussed in Supplementary Information.

Obviously, the states produced with this protocol contain at most n photons. For example, for $n=2$, $|\phi_2\rangle=\sqrt{2/3}|2\rangle-\sqrt{1/3}|0\rangle$. It is quite easy to see that we can prepare 'even' or 'odd' cat states, containing only even or only odd photon numbers, depending on the parity of n. Indeed, in this case the homodyne detection performs a parity measurement: an outcome $p=0$ tells us that the number of photons in the measured mode was even, as the overlap between the (non-physical) state $|p=0\rangle$ and an odd photon number state is null. Therefore, the prepared state has the same parity as n. The homodyne measurement also induces a phase dependence on the originally phase-invariant state. The 3 dB of squeezing required to 'unsqueeze' a cat with $|\alpha|^2=n$ compensate for the loss of half of the photons on the 50/50 BS without changing the parity. Another way to intuitively deduce the scaling law $|\alpha|^2=n$ is to notice that $\phi_n(x)$ presents two peaks centred at $x=\pm\sqrt{n}$, whereas for an ideal cat squeezed by $s_c=1/2$ (3 dB) the same two peaks are at $x=\pm\sqrt{2s_c|\alpha|^2}$. In fact, we can also prepare a cat with a slightly different size by changing s_c if we preserve the relationship $n=2s_c|\alpha|^2$. For example, a odd cat with $|\alpha|^2=9.5$ can be prepared with a 99.7% fidelity using 9 photons with $s_c=9/19$. For an even cat we would use 10 photons and 'unsqueeze' by $s_c=10/19$. Therefore, all parities and cat sizes are accessible.

We have thus proposed and experimentally demonstrated a protocol that allows the preparation of quantum superpositions of squeezed coherent states. Considering the fast technical progress and the increasing number of groups working in this field, we expect that

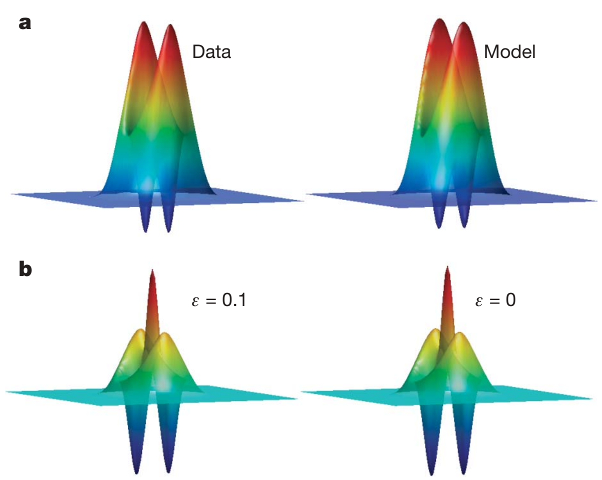

Figure 5 | Influence of experimental imperfections. a, Experimental Wigner function (left) produced from two-photon Fock states, compared to that obtained with our model (right). **b**, Wigner function produced from pure two-photon Fock states with $\varepsilon=0.1$ and no technical imperfections (left) compared to the ideal case with $\varepsilon=0$ (right).

the purity of these superpositions will rapidly improve. The use of higher parametric gains combined with number-resolving photon counters allows the preparation of higher photon number states[27], and should give access to even larger 'Schrödinger cats'. This simple and flexible procedure is particularly suitable for producing these states as 'ancillas' for numerous quantum information processing tasks.

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Supplementary Information is linked to the online version of the paper at www.nature.com/nature.

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